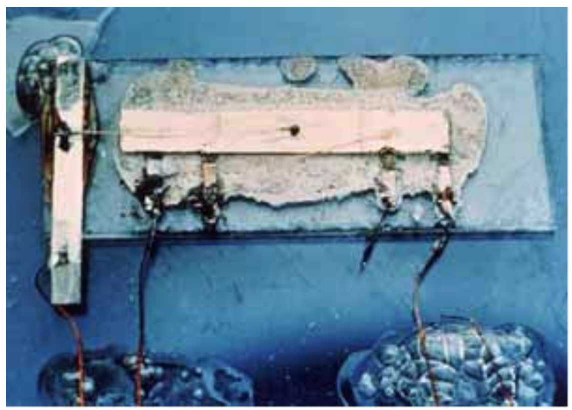
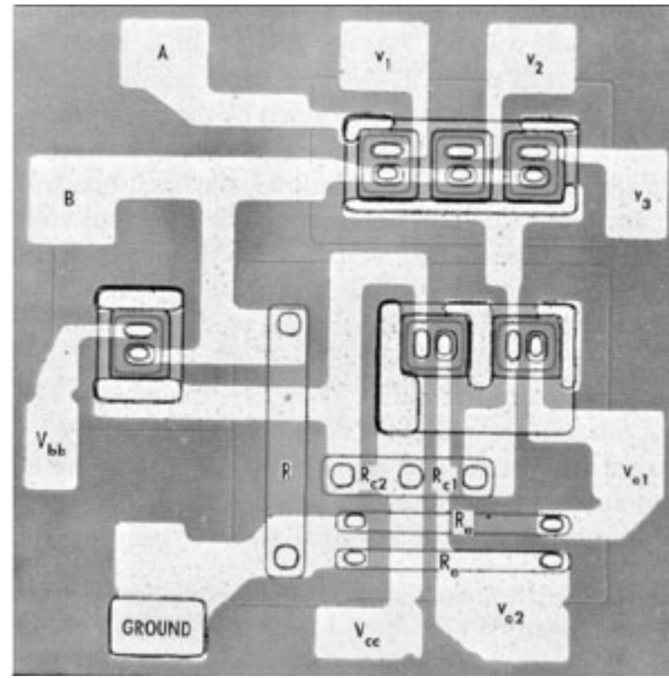
7.5, The Coupled Diode (Ebers-Moll Eqs.)

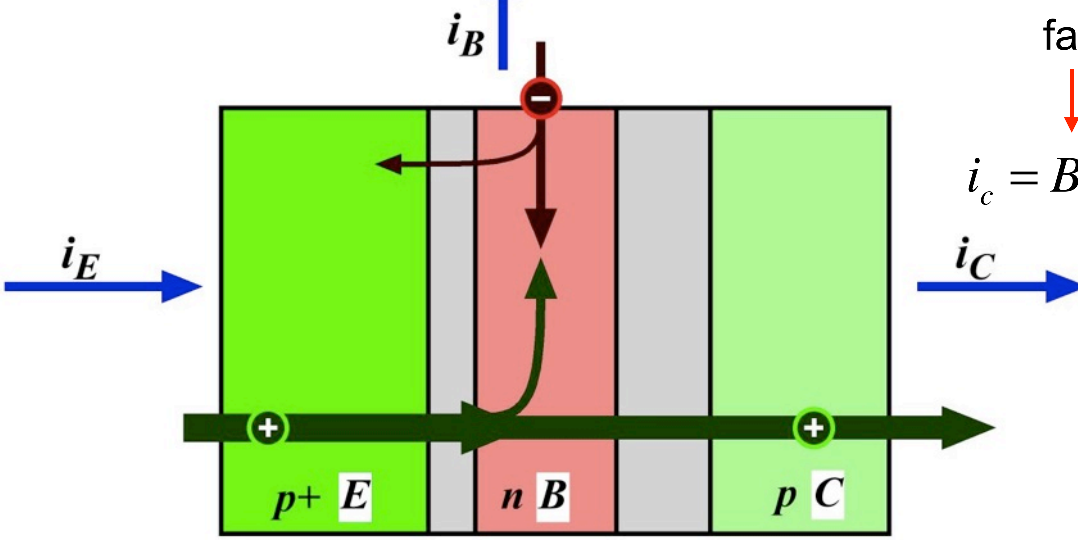
Images are from Texas Instruments, 1958, what are they?





Jack Kilby at TI, developed (at least reported) the 1st IC using BJT technology (differential amplifier, 5 transistors). At nearly same time Robert Noyce at Fairchild also developed one.

$$i_B = i_{En} + (1 - B)i_{Ep}$$

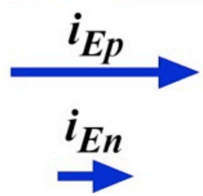


base transport factor

$$i_c = Bi_{Ep}$$

emitter injection efficiency

$$\gamma = \frac{i_{Ep}}{i_{En} + i_{Ep}}$$



current transfer ratio

$$\alpha = \frac{i_C}{i_E} = \frac{Bi_{Ep}}{i_{En} + i_{Ep}} = B\gamma$$

amplification factor

$$\frac{i_C}{i_B} = \frac{B\gamma}{1 - B\gamma} = \frac{\alpha}{1 - \alpha} \equiv \beta$$

$$\Delta p_E = p_n(e^{qV_{EB}/kT} - 1)$$

$$I_{Ep} \approx qA\frac{D_p}{L_p}\,\Delta p_E\,\mathbf{ctnh}\frac{W_b}{L_p}$$

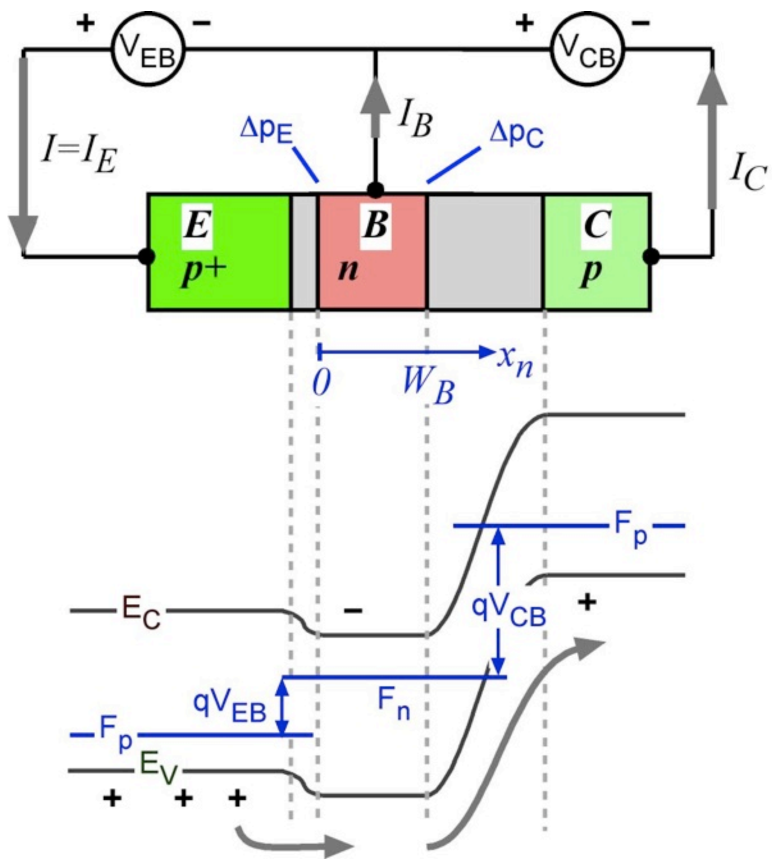
$$I_C \approx qA\frac{D_p}{L_p}\,\Delta p_E\,\mathbf{csch}\frac{W_b}{L_p}$$

$$I_B \approx qA\frac{D_p}{L_p}\,\Delta p_E\,\mathbf{tanh}\frac{W_b}{2L_p}$$

Reverse saturation…

Diode Behavoir…

Ratio’s (10’s-1000’s)



- So you are a BJT device designer or a BJT IC designer… or your boss requests that you do so in 1 week, there should be *simpler* ways than this to design your BJT into circuits for various applications… how?

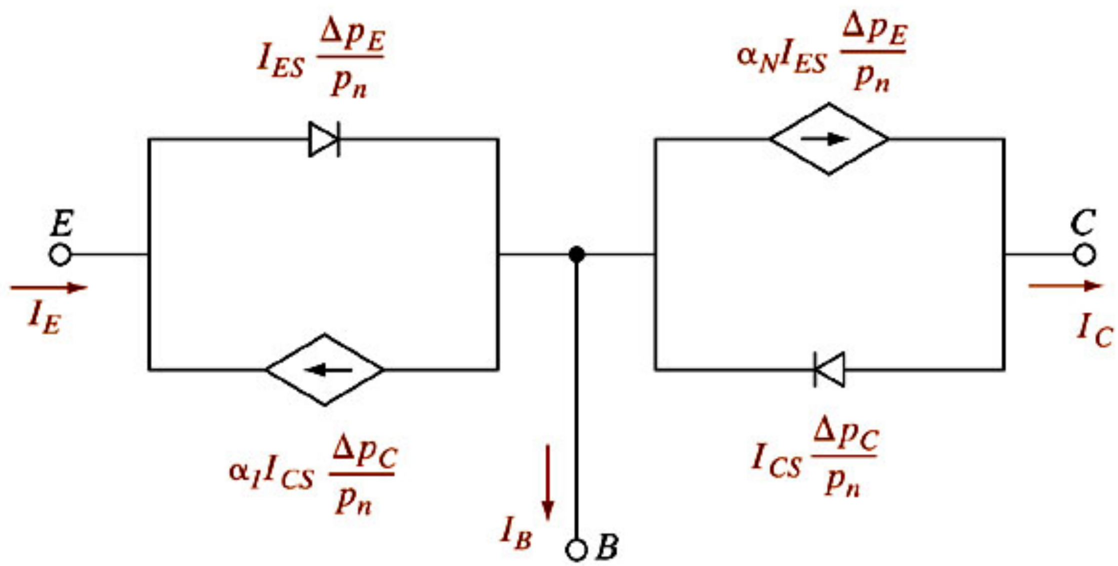
- Furthermore, up to this point we mostly assumed EB strong forward and BC strong reverse… for alternate biasing cases **should we repeat all the derivations again?** Or develop a flexible **simple** circuit model?

- Lets develop the 'Coupled Diode Model' for a BJT that can be biased in more than one way (a.k.a. Ebers-Moll equations). ***There are many models, we'll do only one.***

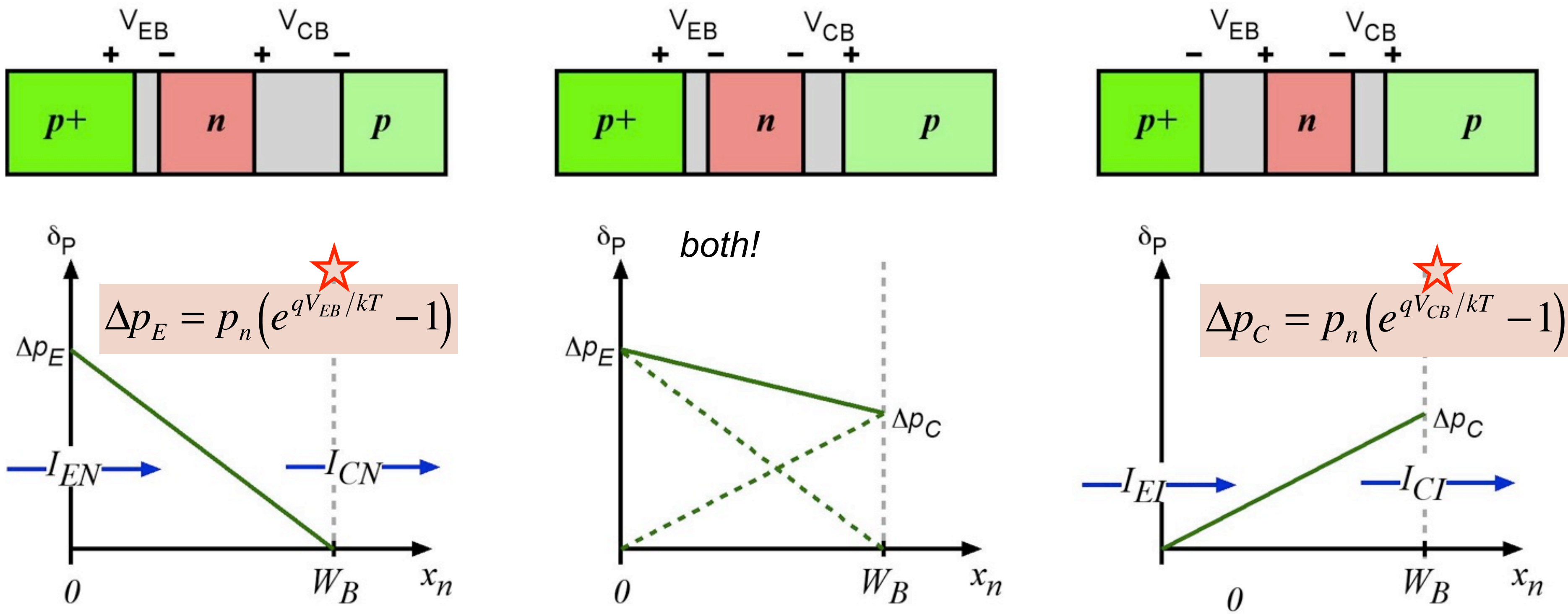
- Why is called 'Coupled Diode'? Because we can solve the case for a pn and np diode separately, and add the solutions to get the simple circuit model for pnp BJT.

First, lets look at base region under some conditions we have not yet considered… and define some new terms…

Once again, it will get very complicated then… then become very simple…!



$$I_B = (1-\alpha_N)\, I_{ES}\frac{\Delta p_E}{p_n} + (1-\alpha_I)\, I_{CS}\frac{\Delta p_C}{p_n}$$

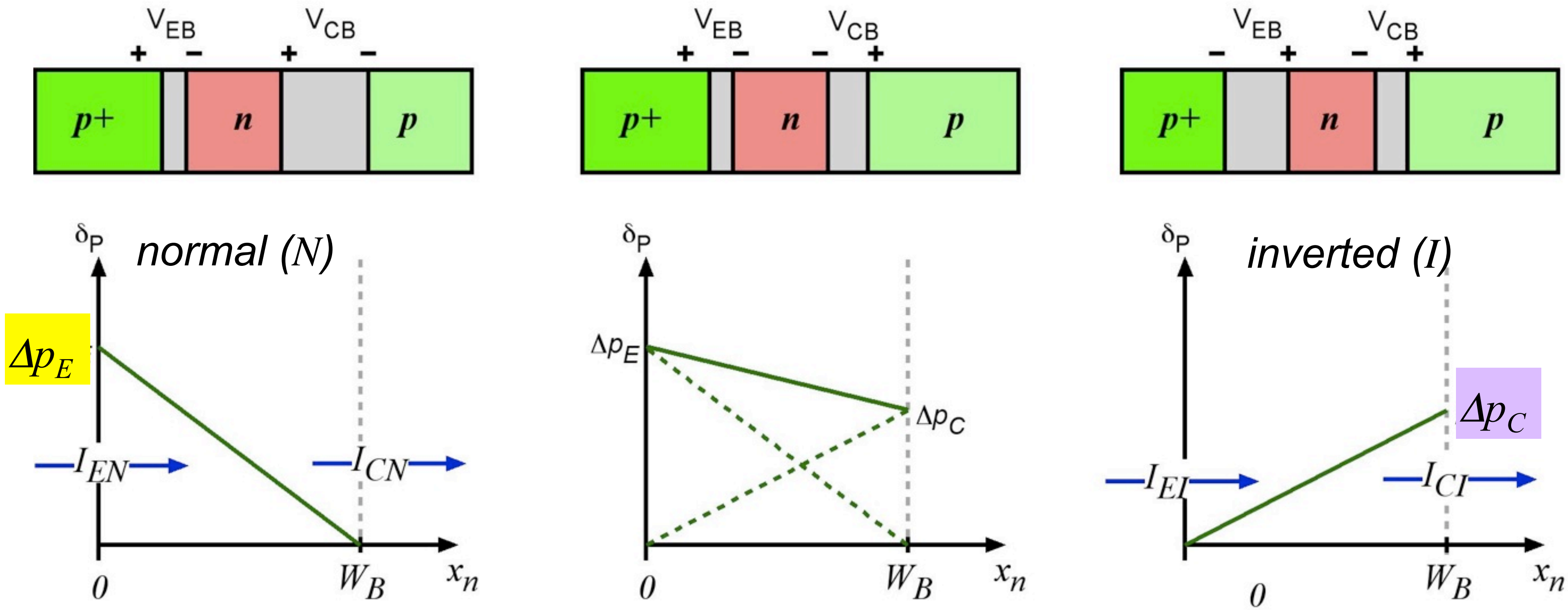


- First note that the solutions for EB or CB hole injections can be linearly super-imposed on each other. Note the voltages and depletion regions in the diagrams above.

- Subscript (N) refers to 'Normal Mode' and (I) to 'Inverted Mode'.

- Note Inverted Mode current flow consistent with Normal Mode direction (therefore has a negative value).

1) Why the excess at one end and zero at other end?
2) Again, what drives current across the base?
3) Lastly, why is CB forward hole injection less than EB?



- We previously showed:

$$I_{Ep} \approx qA\frac{D_p}{L_p}\Delta p_E\ \mathbf{ctnh}\frac{W_b}{L_p}$$

$$I_C \approx qA\frac{D_p}{L_p}\Delta p_E\ \mathbf{csch}\frac{W_b}{L_p}$$

- If we assume geometricaly symmetric then we can write:

N

$$I_{EN} = a\ \Delta p_E$$

$$I_{CN} = b\ \Delta p_E \quad \text{with } \Delta p_C = 0$$

I

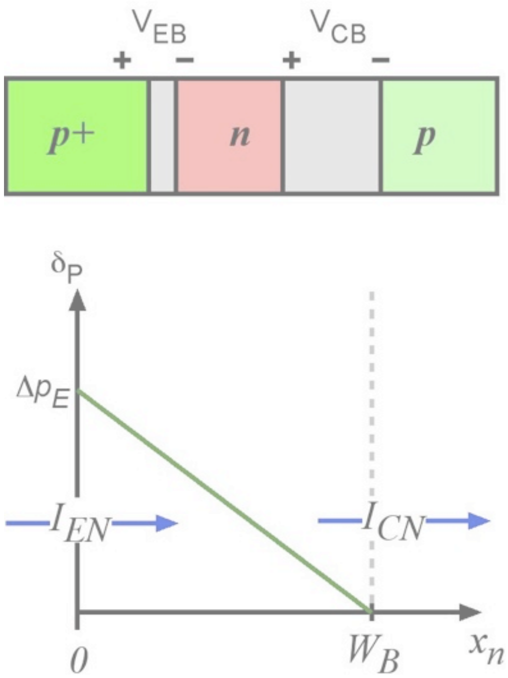
$$I_{EI} = -b\ \Delta p_C$$

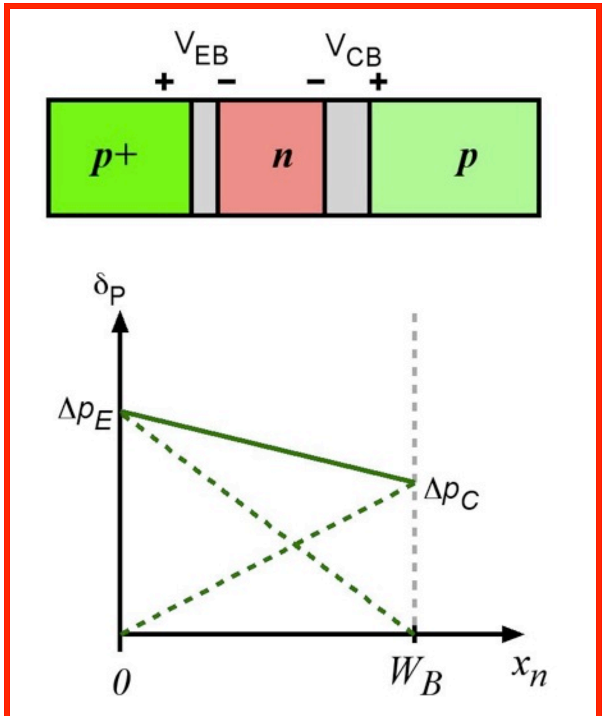
$$I_{CI} = -a\ \Delta p_C \quad \text{with } \Delta p_E = 0$$

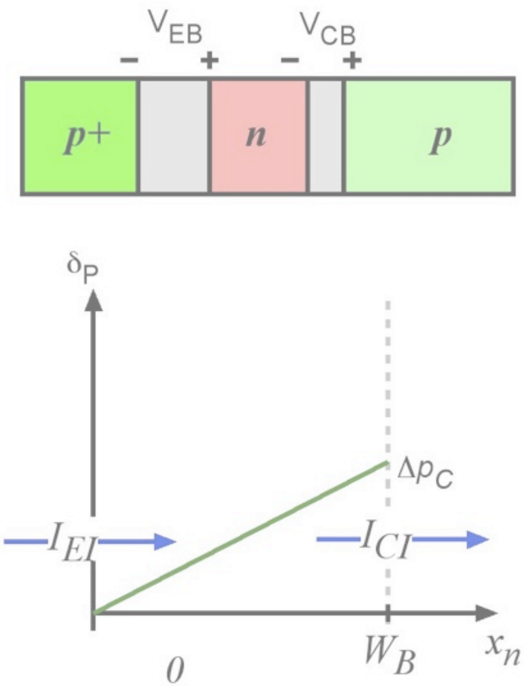
$$a \equiv qA\frac{D_p}{L_p}\text{ctnh}\frac{W_b}{L_p}$$

$$b \equiv qA\frac{D_p}{L_p}\text{csch}\frac{W_b}{L_p}$$

You should understand why we switch the sign, and the 'a's and the 'b's for inverted!







- Remember, we can add by superposition and include Normal and Inverted:

$$I_E = I_{EN} + I_{EI} = a\Delta p_E - b\Delta p_C$$
$$= \mathbf{A}\left(e^{qV_{EB}/kT} - 1\right) - \mathbf{B}\left(e^{qV_{CB}/kT} - 1\right)$$

$$I_C = I_{CN} + I_{CI} = b\Delta p_E - a\Delta p_C$$
$$= \mathbf{B}\left(e^{qV_{EB}/kT} - 1\right) - \mathbf{A}\left(e^{qV_{CB}/kT} - 1\right)$$

where A's and B's are just I_0's...

$$\mathrm{A} \equiv ap_n \quad \text{and} \quad \mathrm{B} \equiv bp_n$$

$$a \equiv qA\frac{D_p}{L_p}\operatorname{ctnh}\frac{W_b}{L_p} \quad \textit{and} \quad b \equiv qA\frac{D_p}{L_p}\operatorname{csch}\frac{W_b}{L_p}$$

however this is for a SYMMETRIC BJT… but our doping levels not equal for E vs. C !!!

- How can we account for asymmetric case (p+np)?

$$I_E = \mathbf{A}\left(e^{qV_{EB}/kT} - 1\right) - \mathbf{B}\left(e^{qV_{CB}/kT} - 1\right)$$

$$I_C = \mathbf{B}\left(e^{qV_{EB}/kT} - 1\right) - \mathbf{A}\left(e^{qV_{CB}/kT} - 1\right)$$

can't account for asymmetry here… why?

$$a \equiv \left(\frac{qAD_p}{L_p}\right)\mathbf{ctnh}\frac{W_b}{L_p} \quad \mathbf{and} \quad b \equiv \left(\frac{qAD_p}{L_p}\right)\mathbf{csch}\frac{W_b}{L_p}$$

$$\mathbf{A} \equiv ap_n \quad \mathbf{and} \quad \mathbf{B} \equiv bp_n$$

can't account for asymmetry here… why?

*However, reverse saturation currents (A, B) need **not** be same for EB and CB!*

- Asymmetric: account for **different rev. sat. currents for each junction** ☆

$$I_{EN} = I_{ES}\left(e^{qV_{EB}/kT} - 1\right), \quad \Delta p_C = 0$$

$$I_{CI} = -I_{CS}\left(e^{qV_{CB}/kT} - 1\right), \quad \Delta p_E = 0$$

the point is $I_{ES} \neq I_{CS}$

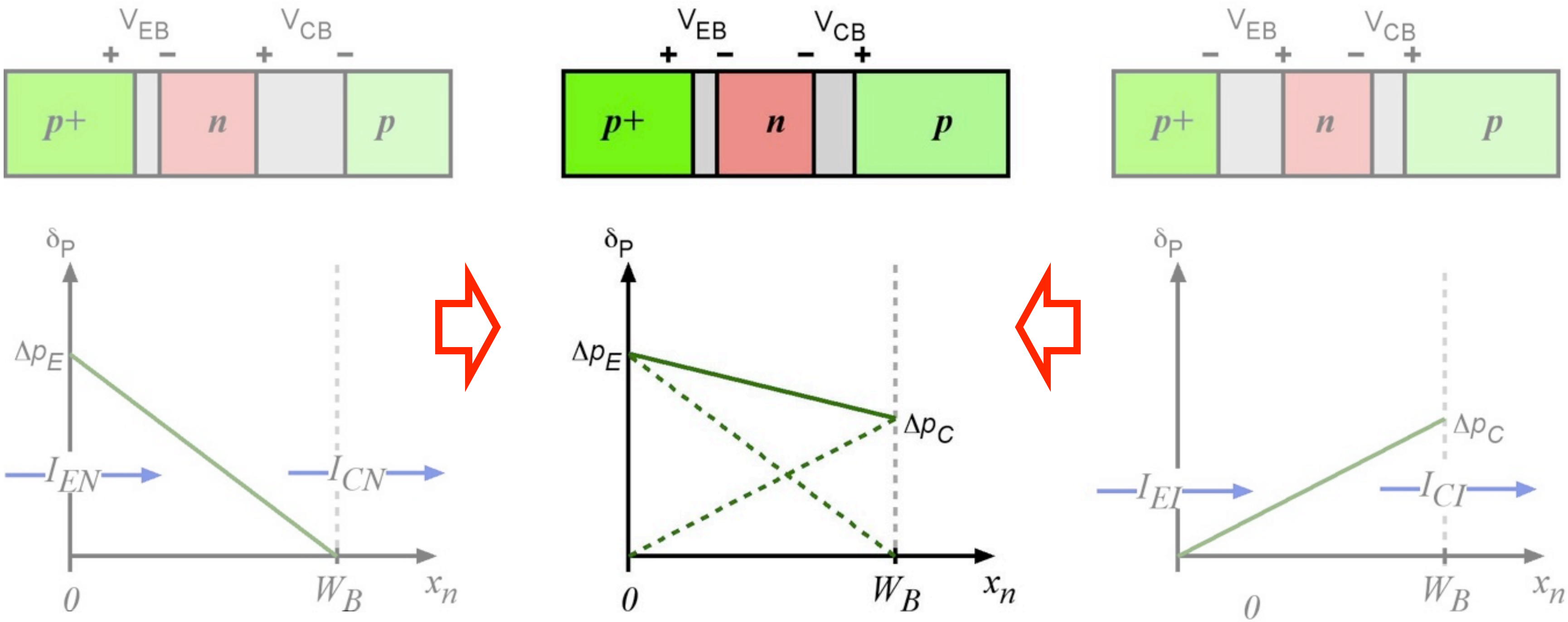
- Asymmetric: therefore **different current transfer ratios** also… ☆

$$I_{CN} = \alpha_N I_{EN} = \alpha_N I_{ES}\left(e^{qV_{EB}/kT} - 1\right)$$

$$I_{EI} = \alpha_I I_{CI} = -\alpha_I I_{CS}\left(e^{qV_{CB}/kT} - 1\right)$$

*now we have all **4** equations… assymetric!*

the point is $\alpha_N \neq \alpha_I$



- Lets add our **4** results by superposition again… we get the famous Ebers-Moll eqs.:

$$I_E = I_{EN} + I_{EI} = I_{ES}\left(e^{qV_{EB}/kT} - 1\right) - \alpha_I I_{CS}\left(e^{qV_{CB}/kT} - 1\right)$$

$$I_C = I_{CN} + I_{CI} = \alpha_N I_{ES}\left(e^{qV_{EB}/kT} - 1\right) - I_{CS}\left(e^{qV_{CB}/kT} - 1\right)$$

Furthermore
(but we will not prove it)

$$\alpha_N I_{ES} = \alpha_I I_{CS}$$

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- We can rewrite using these (what are they again?)

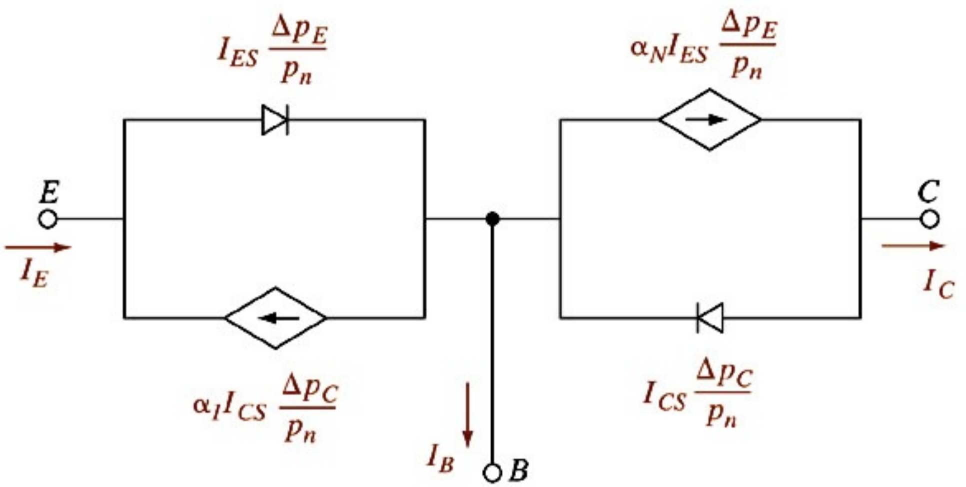
$$\Delta p_E = p_n\left(e^{qV_{EB}/kT}-1\right) \qquad \Delta p_C = p_n\left(e^{qV_{CB}/kT}-1\right)$$

$$I_E = I_{EN} + I_{EI} = I_{ES}\left(e^{qV_{EB}/kT}-1\right) - \alpha_I I_{CS}\left(e^{qV_{CB}/kT}-1\right)$$

$$I_E = I_{ES}\frac{\Delta p_E}{p_n} - \alpha_I I_{CS}\frac{\Delta p_C}{p_n} = \frac{I_{ES}}{p_n}(\Delta p_E - \alpha_N \Delta p_C)$$

$$I_C = I_{CN} + I_{CI} = \alpha_N I_{ES}\left(e^{qV_{EB}/kT}-1\right) - I_{CS}\left(e^{qV_{CB}/kT}-1\right)$$

$$I_C = \alpha_N I_{ES}\frac{\Delta p_E}{p_n} - I_{CS}\frac{\Delta p_C}{p_n} = \frac{I_{CS}}{p_n}(\alpha_I \Delta p_E - \Delta p_C)$$



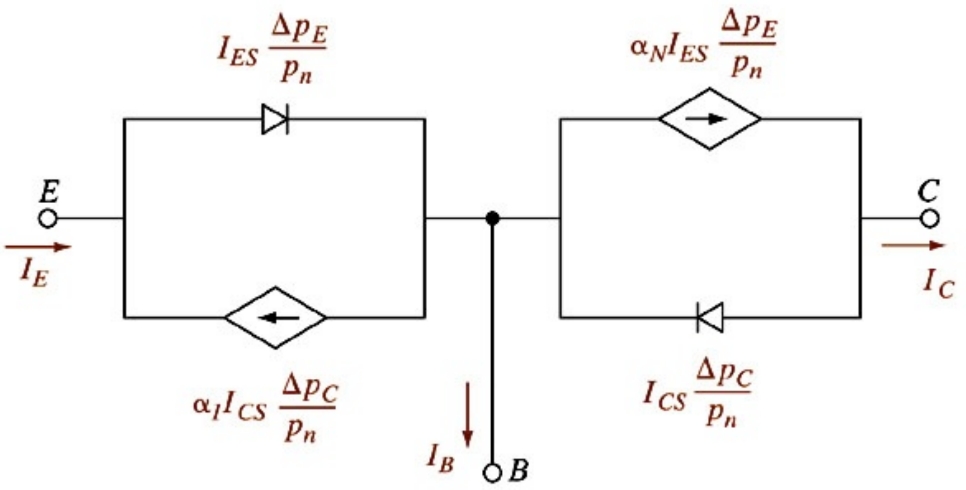
- The whole point of deriving the coupled diode model is so we can put a BJT in a circuit using even simpler circuit components than a BJT (see diagram at right), and, because we can now bias in the circuit in any direction! True or false?

- For a p+np BJT, reverse saturation currents for normal and inverted modes are the: same, or different?

- For a p+np BJT, current transfer ratios for normal and inverted modes are the: same, or different?

- The Δp_C term in the diagram at right, is dependent on: Veb, Vcb, or Vce?

- For a symmetrically doped pnp BJT (emitter and collector doping the same), the reverse saturation currents, and the transfer ratios for normal and inverted modes are the: same, or different?



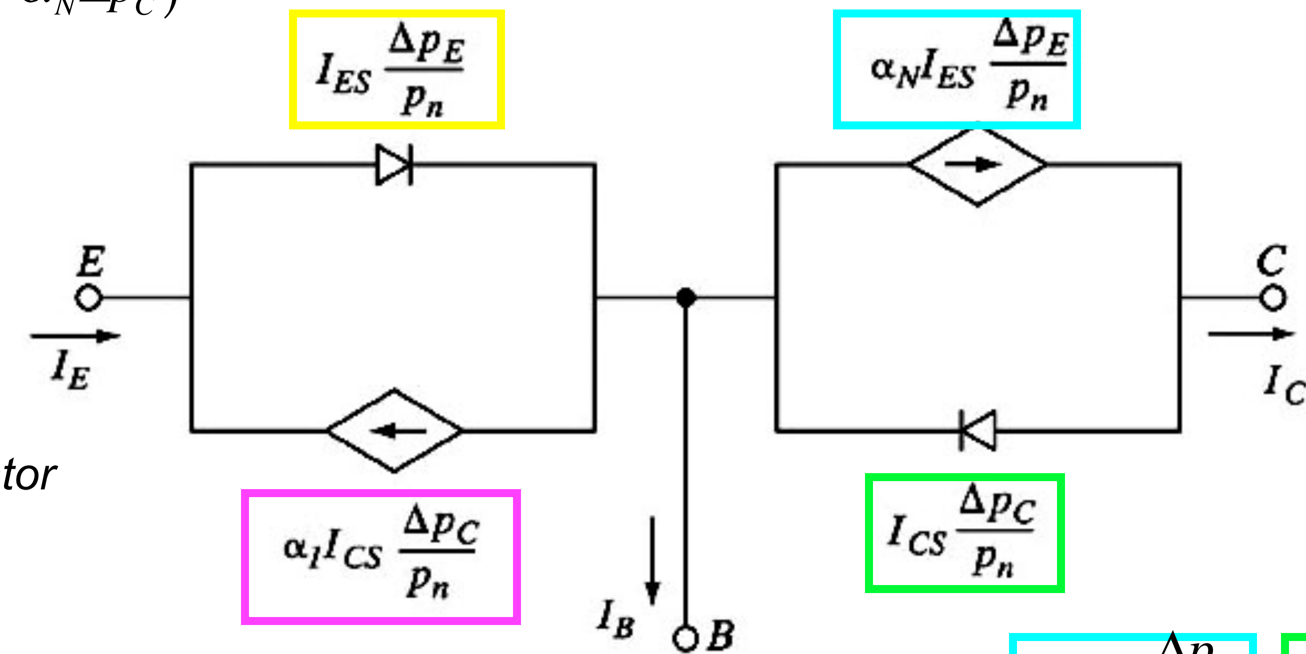
$$I_E = I_{ES}\frac{\Delta p_E}{p_n} - \alpha_I I_{CS}\frac{\Delta p_C}{p_n}$$

$$= \frac{I_{ES}}{p_n}\left(\Delta p_E - \alpha_N \Delta p_C\right)$$

$$\Delta p_E = p_n\left(e^{qV_{EB}/kT} - 1\right)$$

- *Easy to get terminal currents, just sum them up! (follow colors)*

- *Why current sources? Think about how BJT works…*
- *Why the extra p_n in the denominator of diodes?*



$$I_C = \alpha_N I_{ES}\frac{\Delta p_E}{p_n} - I_{CS}\frac{\Delta p_C}{p_n}$$

$$= \frac{I_{CS}}{p_n}\left(\alpha_I \Delta p_E - \Delta p_C\right)$$

$$I_B = (1-\alpha_n)I_{ES}\frac{\Delta p_E}{p_n} + (1-\alpha_I)I_{CS}\frac{\Delta p_C}{p_n}$$

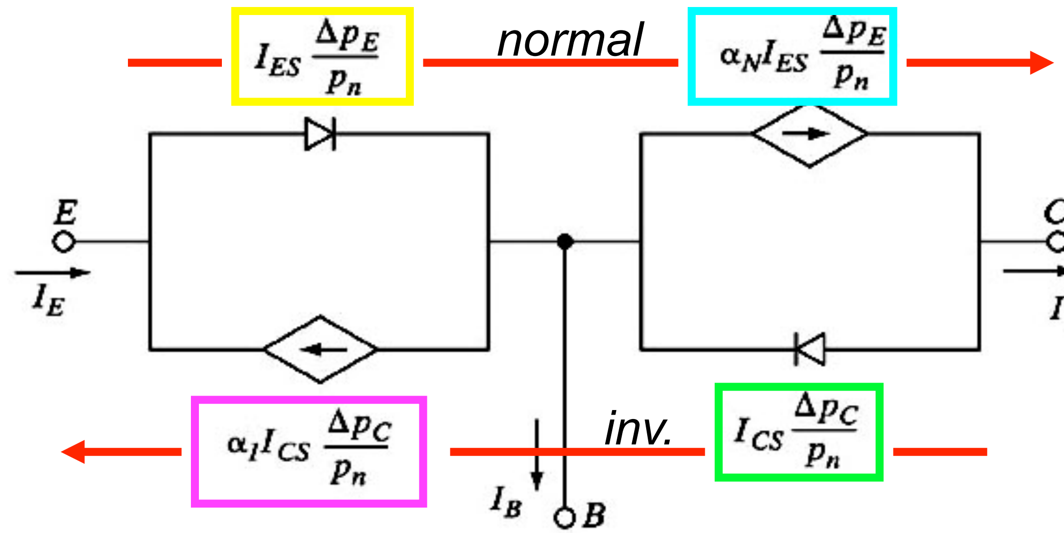
$$\Delta p_C = p_n\left(e^{qV_{CB}/kT} - 1\right)$$

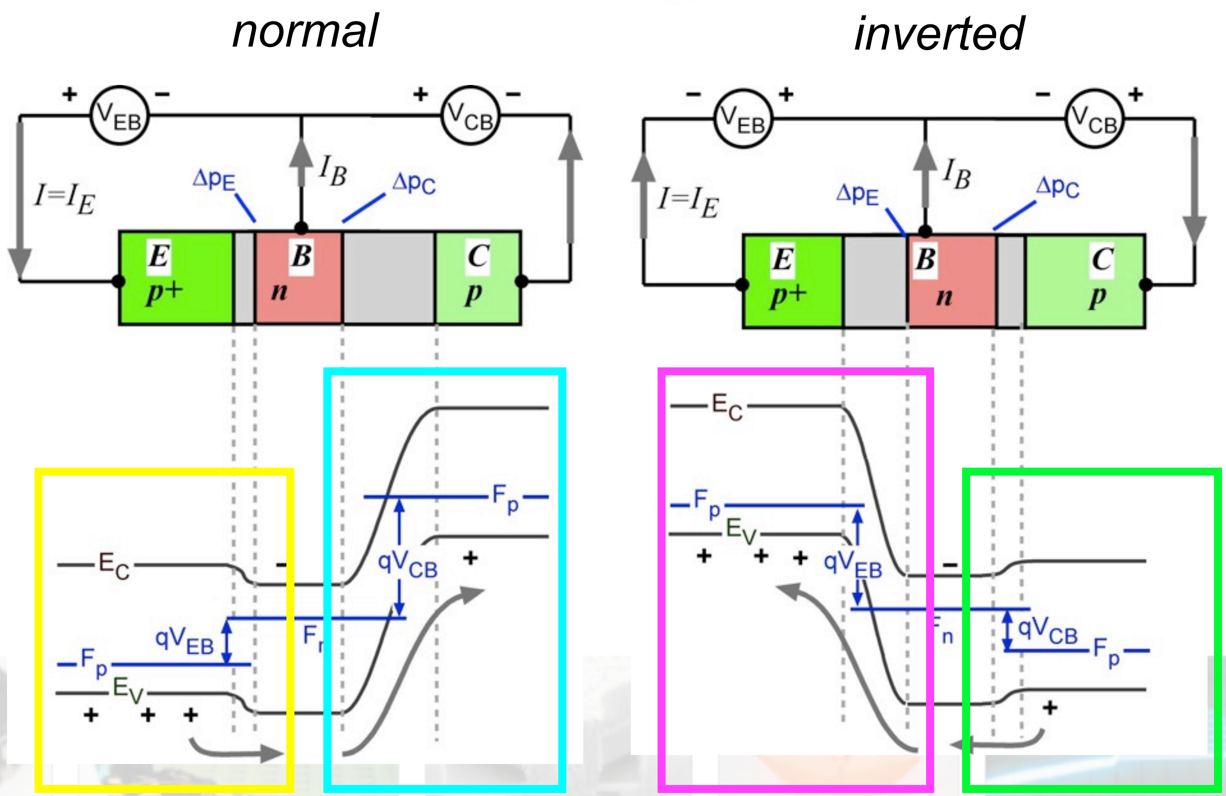
- Now, you can wire a BJT in any fashion you desire, and favorably impress your boss!

- Clever analog circuit designers leverage this! More flexible than a FET!

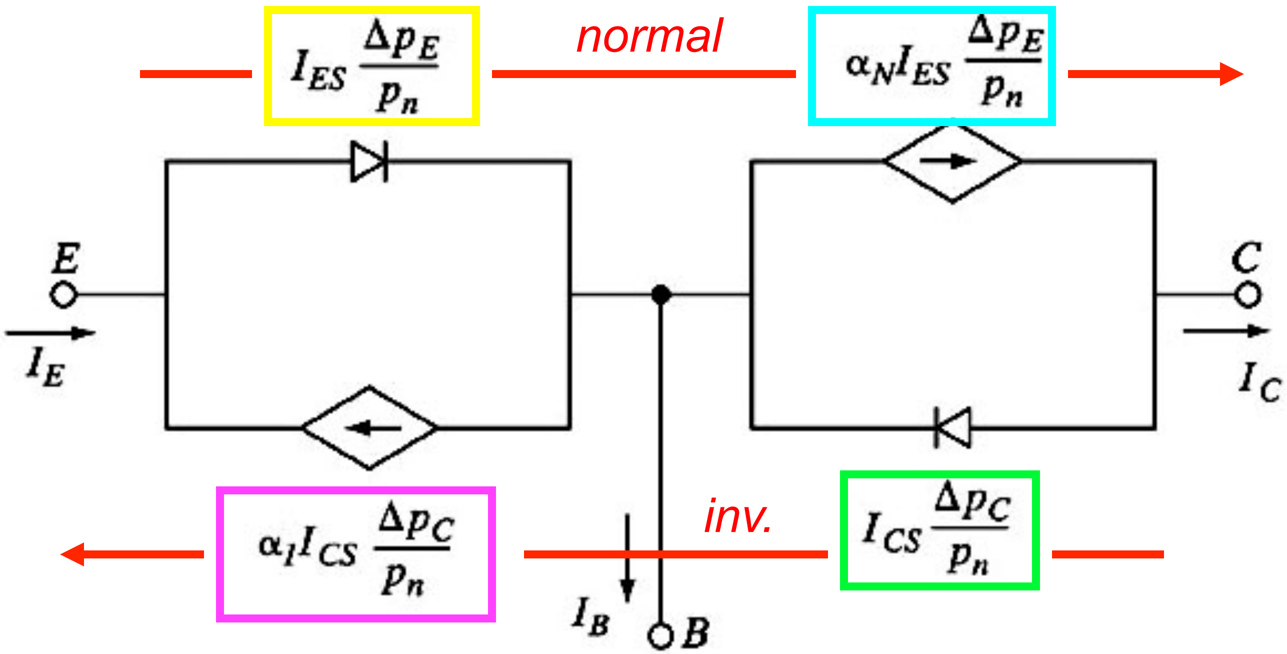
- Remember, all the building blocks are based on CH5 diode behavior

… *see band diagrams for normal and inverted at right*





Again, they are just diodes!



Again, current sources have nothing to do with voltage across that particular junction! Make's sense since reverse bias, I_{drift} is indep. of voltage).

Again, yellow and green boxes really are simple diodes!

Example for 'yellow' box:
1) recall $I_{ES} \sim I_0$
2) remember $\Delta p_C = 0$ in normal mode

$$\Delta p_E = p_n\left(e^{qV_{EB}/kT} - 1\right)$$

$$\Delta p_C = 0$$

normal inv.

$$I_E = I_{ES}\frac{\Delta p_E}{p_n} - \alpha_I I_{CS}\frac{\Delta p_C}{p_n}$$

$$I_E = I_{ES}\frac{\Delta p_E}{p_n} = I_{ES}\left(e^{qV_{EB}/kT} - 1\right)$$

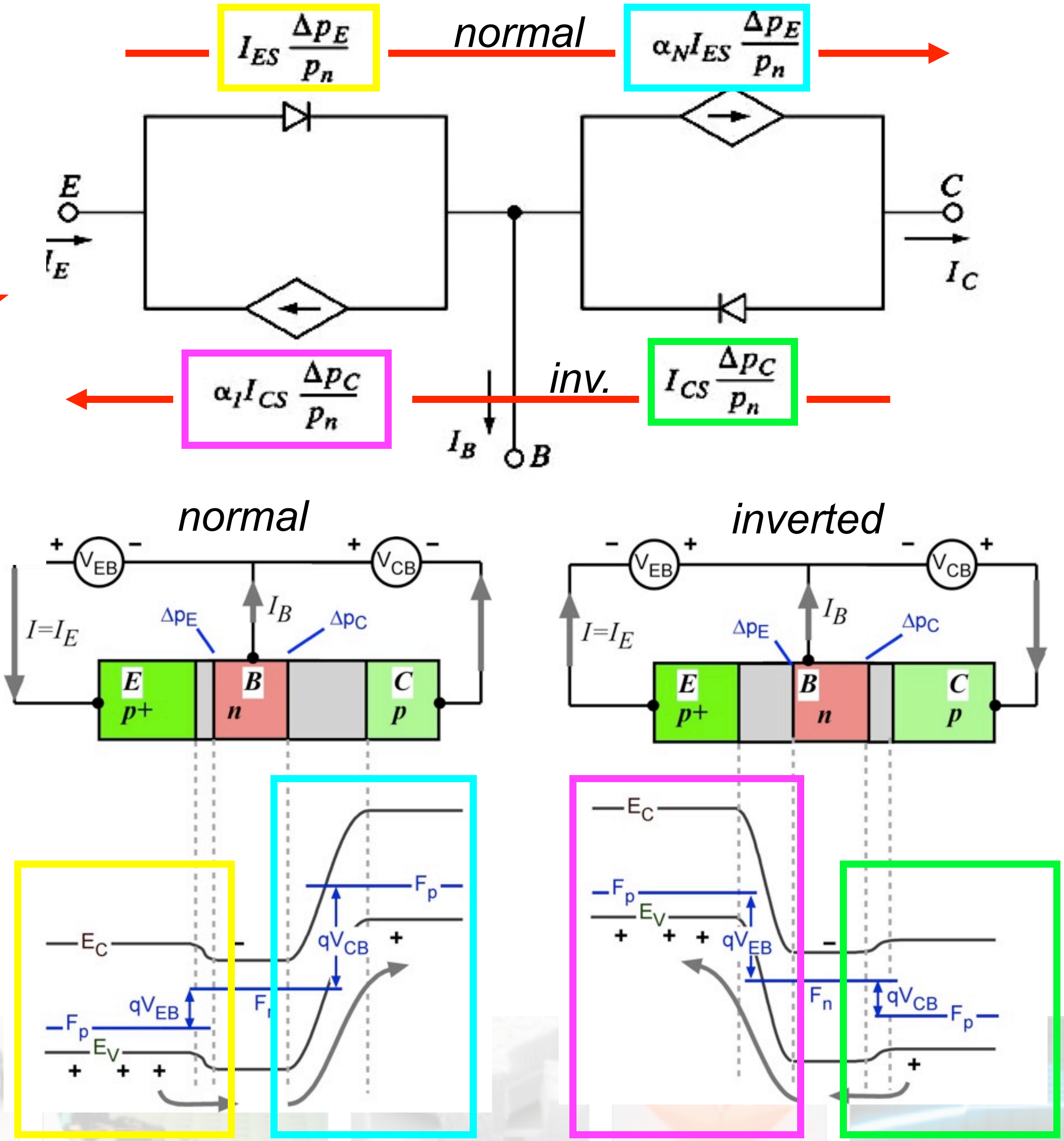
TRUST THE MODEL!

- What would I_B,I_E,I_C be if:

-both EB & CB reverse bias?

-tougher question, would I_B be larger or small if symmetric pnp both EB & CB were in forward bias? What does the model reveal?

- *1st no net h concentration gradient across the base so nothing to drive I_C or I_E,*
- *2nd, W is narrow, so very little recombination, so I_B would be very small (negligible in some circuit designs).*



- There are alternate representations… you can look at alt. circuit model in 7-11, where we can get new terms needed for the circuit as follows:

- Eliminate the saturation current from the equation by multiplying the emitter current equations by α_N and then subtracting the collector current to get:

$$I_E = I_{ES}\frac{\Delta p_E}{p_n} - \alpha_I I_{CS}\frac{\Delta p_C}{p_n} = \frac{I_{ES}}{p_n}(\Delta p_E - \alpha_N \Delta p_C)$$

$$I_E = \alpha_I I_C + I_{EO}\left(e^{qV_{EB}/kT} - 1\right)$$

where $I_{EO} = (1 - \alpha_N \alpha_I) I_{ES}$

- Similarly for collector current:

$$I_C = \alpha_N I_{ES}\frac{\Delta p_E}{p_n} - I_{CS}\frac{\Delta p_C}{p_n} = \frac{I_{CS}}{p_n}(\alpha_I \Delta p_E - \Delta p_C)$$

$$I_C = \alpha_N I_E - I_{CO}\left(e^{qV_{CB}/kT} - 1\right)$$

where $I_{CO} = (1 - \alpha_N \alpha_I) I_{CS}$

- Does the model have to be symmetrically doped for the emitter and collector (same doping levels)? *Hint, two things in the model at right will depend on doping, what terms are they?*

- Why current sources? *Hint, think of the basic definition of a current source, and note how they depend on Δp_E and Δp_C and the voltages inside those are from another part of the circuit.*

- If V_{EB} and V_{CB} are both positive, what will I_B be and why? Very large, very small/zero, or the model does not provide this info.

- How would you change the model if W_B became really large? *Hint, would it still be a BJT? If not, then you eliminate two of the four circuit components in the model, which ones?*

- Would the specific BJT shown at right run better (higher amplification) in normal mode, inverted mode, or will they be the same for both modes? *Hint, look at emitter and collector dopings…*

- If you fed in all the terms for the boxes in yellow and green, what would the equation reduce to? A diode, a current source, or can't be solved?